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In this letter we address the question of the phenomena of Andreev reflection between the cold quark-gluon plasma phase and CFL color superconductor. We show that there are two different types of reflections connected to the structure of the CFL phase. We also calculate the baryon current at the interface and we show that it vanishes for energy of scattering quarks below the superconducting gap.

I. INTRODUCTION

The prediction of the existence of the superconducting phase as a true vacuum for high density QCD [1] raises the interesting possibility that this phase can be found inside the Neutron Stars. There are a lot of papers which try to find the possible observable effects that can be caused by the superconducting phase (see for review [2]).

In this letter we try to point out on the phenomenon that take place at the interface between the quark-gluon plasma and CFL phase [3] which can be of great importance for transport phenomena of baryonic matter inside the Neutron Star. There is a well-known process of Andreev reflection [4] in condensed matter systems that can also take place in the case of color superconductors. This possibility was mentioned in the paper [5] for the case of two light flavor superconductivity (2SC phase). Here we consider in detail the Andreev reflection process for CFL superconductors.

In the second section we describe the effective hamiltonian and later we discuss the influence of Nambu-Goldstone modes for the reflection process. The fourth section contains the detail discussion of Andreev reflection and formula for probability current.

II. EFFECTIVE HAMILTONIAN

In QCD at asymptotically high density, the dominant interaction between quarks is carried by one gluon exchange, in which the interaction force is attractive in the color $\bar{3}$ channel. The dominant coupling is a color and flavor anti-symmetric interaction of the form from one gluon exchange [6]

$$\mathcal{L}_{eff} = G(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc})(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})(\psi_i^{aT}C\gamma_5\psi_j^b)(\psi_k^{cT}C\gamma_5\psi_l^d)^\dagger. \quad (1)$$

where a, b, c and d are color indices and i, j, k and l are flavor indices. ψ_i^a is the quark field operator.

Now let us assume that the condensate takes the form

$$4G < \psi_i^{aT}C\gamma_5\psi_j^b > = \Delta_{ij}^{ab}. \quad (2)$$

Here Δ_{ij}^{ab} is anti-symmetric under the exchanges of $a \leftrightarrow b$ and $i \leftrightarrow j$, respectively.

Using eq.(2) one obtains at the mean field level:

$$\mathcal{L}_{eff} = (\Delta_{ij}^{ab}(\psi_k^{cT}C\gamma_5\psi_l^d)^\dagger + h.c.) + \tilde{\mathcal{L}}. \quad (3)$$

where $\tilde{\mathcal{L}}$ is independent of the fermionic fields.

The effective hamiltonian of the system of interest is

$$H_{eff} = \int d^3x \left[\psi_i^{a\dagger}(-i\vec{\alpha} \cdot \vec{\nabla} + m\gamma_0 - \mu)\psi_i^a + (\Delta_{ij}^{ab}(\psi_i^{aT}C\gamma_5\psi_j^b)^\dagger + h.c.) \right] \quad (4)$$

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with $\vec{\alpha} \equiv \gamma_0 \vec{\gamma}$. Here m is the current quark mass and μ is the quark chemical potential. Δ depends on the positions: it vanishes in the QGP phase and takes non-zero value in CFL phase.

In CFL phase, color and flavor degrees of freedom are locked to each other. So the gap matrix Δ_{ij}^{ab} , which is a 3×3 matrix in both color and flavor spaces, takes the form [3]

$$\Delta_{ij}^{ab} = \Delta(\delta_i^a \delta_j^b - \delta_j^a \delta_i^b). \quad (5)$$

Here we have neglected the contribution from color 6 channel, which is symmetric in color and flavor since it was shown to be small [3].

From the structure of the gap matrix (5), we find that there are two kinds of quark pair structures. One consists of only two quark contribution, such as

$$\begin{pmatrix} u_{green} \\ d_{red} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} u_{blue} \\ 0 \\ s_{red} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ d_{blue} \\ s_{green} \end{pmatrix}. \quad (6)$$

These cases are similar to 2SC phase, where we have only two flavors. On the other hand in the CFL phase there is another structure of quark pairs composed of

$$\begin{pmatrix} u_{red} \\ d_{green} \\ s_{blue} \end{pmatrix}, \quad (7)$$

which never appears in 2SC phase¹. Let us call it the triplet for short. We will mainly restrict our consideration of Andreev reflection into this case because the 2SC-like pairs have been already discussed in [5].

III. NAMBU-GOLDSTONE MODES

Before going into details, let us mention about another aspect of CFL phase. Unlike 2SC phase, there exist Nambu-Goldstone (NG) bosons associated with chiral and baryon symmetry breaking in CFL phase. Because they are the lowest order excitations one can think that they can influence in large extent the Andreev reflection process. However we find the NG bosons do not play an important role in the current problem.

The effective interaction in the lowest order between fermions and NG bosons in CFL phase is given by the formula [8]:

$$\begin{aligned} H^{\text{int}} &= H_{\chi\chi\Pi} + H_{\chi\chi\Pi\Pi} \\ &= \chi^\dagger \begin{pmatrix} 0 & \Delta \left(\frac{2i}{F_\Pi} \Pi - \frac{2}{F_\Pi^2} \Pi^2 \right) \\ \Delta^* \left(-\frac{2i}{F_\Pi} \Pi - \frac{2}{F_\Pi^2} \Pi^2 \right) & 0 \end{pmatrix} \chi, \end{aligned} \quad (8)$$

where χ is the Nambu-Gorkov fermion field and Π is the Nambu-Goldstone field. F_Π is the decay constant of the NG field. From this form of the interaction we find the following Feynman rules (Fig.1):

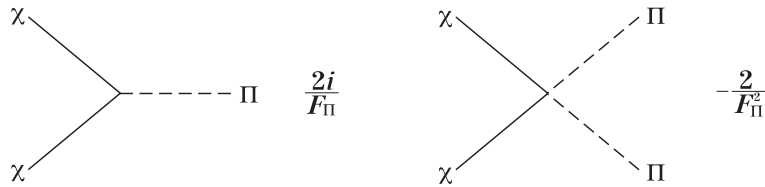


Fig.1 The lowest order interaction vertices between quarks and the NG bosons.

As has been obvious from these diagrams, the interaction between quarks and the NG bosons is suppressed by the factor $\frac{1}{F_\Pi}$. On the other hand, F_Π is of the order of $\mathcal{O}(\mu)$ [9]. As the result, the emission of the NG bosons from quarks is highly suppressed at high density. So we can neglect the effect of the NG bosons at leading order in μ expansion.

¹This quark pair structures have been discussed in [7].

Let us now discuss Andreev reflection at QGP/CFL interface for the triplet state we mentioned in the chapter II. We choose u_{red} , d_{green} and s_{blue} basis because it is natural in the QGP phase. The equations of motions that follow from (4) take the form:

$$\begin{aligned}
 i\dot{\psi}_{red}^u &= (-i\vec{\alpha} \cdot \vec{\nabla} + m\gamma_0 - \mu)\psi_{red}^u - \Delta C\gamma_5(\psi_{green}^{d*} + \psi_{blue}^{s*}), \\
 i\dot{\psi}_{green}^d &= (-i\vec{\alpha} \cdot \vec{\nabla} + m\gamma_0 - \mu)\psi_{green}^d - \Delta C\gamma_5(\psi_{red}^{u*} + \psi_{blue}^{s*}), \\
 i\dot{\psi}_{blue}^s &= (-i\vec{\alpha} \cdot \vec{\nabla} + m\gamma_0 - \mu)\psi_{blue}^s - \Delta C\gamma_5(\psi_{red}^{u*} + \psi_{green}^{d*}), \\
 i\dot{\psi}_{red}^{u\dagger} &= -i\vec{\nabla}\psi_{red}^{u\dagger} \cdot \vec{\alpha} - \psi_{red}^{u\dagger}(m\gamma_0 - \mu) - \Delta^*(\psi_{green}^{dT} + \psi_{blue}^{sT})C\gamma_5, \\
 i\dot{\psi}_{green}^{d\dagger} &= -i\vec{\nabla}\psi_{green}^{d\dagger} \cdot \vec{\alpha} - \psi_{green}^{d\dagger}(m\gamma_0 - \mu) - \Delta^*(\psi_{red}^{uT} + \psi_{blue}^{sT})C\gamma_5, \\
 i\dot{\psi}_{blue}^{s\dagger} &= -i\vec{\nabla}\psi_{blue}^{s\dagger} \cdot \vec{\alpha} - \psi_{blue}^{s\dagger}(m\gamma_0 - \mu) - \Delta^*(\psi_{red}^{uT} + \psi_{green}^{dT})C\gamma_5.
 \end{aligned} \tag{9}$$

To find the quasiparticle wavefunctions for $\Delta = const$, it is convenient to use the following decompositions:

$$\begin{aligned}
 \psi_{red}^u(t, \vec{r}) &= \sum_r \alpha_r \varphi_{r,R}^u(\vec{q}) \exp(i\vec{q} \cdot \vec{r} - iEt), & \psi_{red}^{u\dagger}(t, \vec{r}) &= \sum_r \alpha_r^* h_{r,L}^{u\dagger}(-\vec{q}) \exp(i\vec{q} \cdot \vec{r} - iEt), \\
 \psi_{green}^d(t, \vec{r}) &= \sum_r \beta_r \varphi_{r,R}^d(\vec{q}) \exp(i\vec{q} \cdot \vec{r} - iEt), & \psi_{green}^{d\dagger}(t, \vec{r}) &= \sum_r \beta_r^* h_{r,L}^{d\dagger}(-\vec{q}) \exp(i\vec{q} \cdot \vec{r} - iEt), \\
 \psi_{blue}^s(t, \vec{r}) &= \sum_r \gamma_r \varphi_{r,R}^s(\vec{q}) \exp(i\vec{q} \cdot \vec{r} - iEt), & \psi_{blue}^{s\dagger}(t, \vec{r}) &= \sum_r \gamma_r^* h_{r,L}^{s\dagger}(-\vec{q}) \exp(i\vec{q} \cdot \vec{r} - iEt),
 \end{aligned} \tag{10}$$

where φ and h are defined in Appendix. α_r, β_r and γ_r are some constants. The subscripts u, d and s describe flavor and color of quarks in obvious way.

Plugging (10) into (9), using the bispinor algebraic relations given in Appendix and assuming constant value of the gap parameter Δ , one obtains the wavefunction describing the quasiparticle excitations of given energy E in CFL phase:

$$\begin{aligned}
 \Psi(t, \vec{r}) \equiv \begin{pmatrix} \psi_{red}^u \\ \psi_{green}^d \\ \psi_{blue}^s \\ \psi_{red}^{u\dagger T} \\ \psi_{green}^{d\dagger T} \\ \psi_{blue}^{s\dagger T} \end{pmatrix} &= \left[A \begin{pmatrix} \frac{E+\xi}{\Delta^*} \varphi_{\uparrow R}^u \\ -\frac{E+\xi}{\Delta^*} \varphi_{\uparrow R}^d \\ 0 \\ -h_{\downarrow L}^{u\dagger T} \\ h_{\downarrow L}^{d\dagger T} \\ 0 \end{pmatrix} e^{i\vec{q}_1 \cdot \vec{r}} + B \begin{pmatrix} \frac{E-\xi}{\Delta^*} \varphi_{\uparrow R}^u \\ -\frac{E-\xi}{\Delta^*} \varphi_{\uparrow R}^d \\ 0 \\ -h_{\downarrow L}^{u\dagger T} \\ h_{\downarrow L}^{d\dagger T} \\ 0 \end{pmatrix} e^{i\vec{q}_2 \cdot \vec{r}} + C \begin{pmatrix} \frac{E+\xi}{\Delta^*} \varphi_{\uparrow R}^u \\ 0 \\ -\frac{E+\xi}{\Delta^*} \varphi_{\uparrow R}^s \\ -h_{\downarrow L}^{u\dagger T} \\ 0 \\ h_{\downarrow L}^{s\dagger T} \end{pmatrix} e^{i\vec{q}_1 \cdot \vec{r}} + \right. \\
 &\quad \left. + D \begin{pmatrix} \frac{E-\xi}{\Delta^*} \varphi_{\uparrow R}^u \\ 0 \\ -\frac{E-\xi}{\Delta^*} \varphi_{\uparrow R}^s \\ -h_{\downarrow L}^{u\dagger T} \\ 0 \\ h_{\downarrow L}^{s\dagger T} \end{pmatrix} e^{i\vec{q}_2 \cdot \vec{r}} + F \begin{pmatrix} \frac{2\Delta}{E-\zeta} \varphi_{\uparrow R}^u \\ \frac{2\Delta}{E-\zeta} \varphi_{\uparrow R}^d \\ \frac{2\Delta}{E-\zeta} \varphi_{\uparrow R}^s \\ h_{\downarrow L}^{u\dagger T} \\ h_{\downarrow L}^{d\dagger T} \\ h_{\downarrow L}^{s\dagger T} \end{pmatrix} e^{i\vec{p}_1 \cdot \vec{r}} + G \begin{pmatrix} \frac{2\Delta}{E+\zeta} \varphi_{\uparrow R}^u \\ \frac{2\Delta}{E+\zeta} \varphi_{\uparrow R}^d \\ \frac{2\Delta}{E+\zeta} \varphi_{\uparrow R}^s \\ h_{\downarrow L}^{u\dagger T} \\ h_{\downarrow L}^{d\dagger T} \\ h_{\downarrow L}^{s\dagger T} \end{pmatrix} e^{i\vec{p}_2 \cdot \vec{r}} \right] \exp(-iEt) \tag{11}
 \end{aligned}$$

where $\xi = \sqrt{E^2 - \Delta^2}$ and $\zeta = \sqrt{E^2 - 4\Delta^2}$. $q_{1,2} = \sqrt{(\mu \pm \xi)^2 - m^2}$ and $p_{1,2} = \sqrt{(\mu \pm \zeta)^2 - m^2}$. A, B, C, D, F and G are arbitrary constants. Similar expression is obtained for the opposite spin content.

Let us set up our physical problem of the quark scattering at the QGP/CFL interface. We suppose here that there is plane boundary of the interface at $z = 0$. The boundary is defined as the step function $\Delta(z) = \Delta\Theta(z)$. Then we try to find out the solutions of eqs.(9) by requiring the wavefunctions on both sides (QGP and CFL) to match at $z = 0$.

Let us assume the “red” up quark with given energy E falls at the plane boundary from the left $z < 0$ (QGP phase). Then the wavefunction (for $\Delta = 0$) takes the form:

$$\Psi_{<}(t, z) = \begin{pmatrix} \varphi_{\uparrow R}^u e^{ik_1 z} + H \varphi_{\uparrow R}^u e^{-ik_1 z} \\ J \varphi_{\uparrow R}^d e^{-ik_2 z} \\ K \varphi_{\uparrow R}^s e^{-ik_3 z} \\ L h_{\downarrow L}^{u\uparrow T} e^{ik_4 z} \\ N h_{\downarrow L}^{d\uparrow T} e^{ik_5 z} \\ P h_{\downarrow L}^{s\uparrow T} e^{ik_6 z} \end{pmatrix} \exp(-iEt), \quad (12)$$

where H, J, K, L, N and P denote the amplitudes of the reflection of the particle and holes and $k_1 = k_2 = k_3 \equiv k = \sqrt{(\mu + E)^2 - m^2}$ and $k_4 = k_5 = k_6 \equiv l = \sqrt{(\mu - E)^2 - m^2}$, respectively. For $z > 0$, the quasiparticle excitations in CFL phase are described by the wavefunction $\Psi_{>}(t, z)$ given by the expression (11). The continuity conditions to match the wavefunctions at the interface are of the form:

$$\Psi_{<}(t, z = 0) = \Psi_{>}(t, z = 0). \quad (13)$$

Using this condition one can find the amplitude of the scattering process at leading order in μ expansion in the massless limit:

$$\begin{aligned} A = C &= \frac{\Delta}{3(E + \xi)} + \mathcal{O}\left(\frac{1}{\mu}\right), \\ F &= \frac{2\Delta}{3(E + \zeta)} + \mathcal{O}\left(\frac{1}{\mu}\right), \\ L &= \frac{E - \zeta}{6\Delta} - \frac{2\Delta}{3(E + \xi)} + \mathcal{O}\left(\frac{1}{\mu}\right), \\ N = P &= \frac{E - \zeta}{6\Delta} + \frac{\Delta}{3(E + \xi)} + \mathcal{O}\left(\frac{1}{\mu}\right) \end{aligned} \quad (14)$$

and other coefficients vanish in the limit where $\Delta, E \ll \mu$. It is worth mentioning more about the result obtained here. By the scattering of the red up quark at the interface, holes of the green down quark and the blue strange quark can be reflected into the QGP phase and the quasiparticles with momenta q_1 and p_1 can propagate in the CFL phase. This is the similar property which has been observed in the QGP/2SC interface [5]. However, unlike the QGP/2SC case, hole of the red up quark is also reflected in the QGP phase. This can be interpreted as follows. When the red up quark falls toward the boundary ($z = 0$), it takes another quark together in order to make a Cooper pair. In the 2SC case, the Cooper pair takes the form $\langle ud \rangle$ so the up quark takes only the down quark leaving the d-hole in QGP phase. On the other hand, in the CFL phase, we have two kinds of gaps. One is the gap for octet excitation and the other is for singlet excitation. In the former case, the condensate takes the form $\langle ud \rangle$, $\langle us \rangle$ and $\langle ds \rangle$ similar to the 2SC case. In the latter case, the condensate takes the form $\langle uu - dd - 2ss \rangle$, which means that the up quark has take not only d and s , but also u quarks to make a pair. In this way we are left with these holes of u, d and s -type.

Finally let us calculate the probability current. The conserved probability current is given by $\vec{j} = \Psi_{<}^\dagger \vec{\alpha} \Psi_{<} = \Psi_{>}^\dagger \vec{\alpha} \Psi_{>}$. Using eqs.(11) (or (12)) and (14) one finds:

$$j_z = \begin{cases} 0 & \text{for } E < |\Delta|, \\ 2\mu \frac{4}{3} \frac{\xi}{E + \xi} & \text{for } |\Delta| < E < 2|\Delta|, \\ 2\mu \frac{4}{3} \frac{\xi}{E + \xi} + 2\mu \frac{2}{3} \frac{\zeta}{E + \zeta} & \text{for } E > 2|\Delta|. \end{cases} \quad (15)$$

The result (15) is interpreted as follows: If the incoming up quark has energy below Δ it cannot excite quasiparticles in CFL phase. However if the up quark has energy between $|\Delta|$ and $2|\Delta|$, it excites quasiparticles which are separated from the vacuum by the gap $|\Delta|$. If the up quark possesses energy above $2|\Delta|$ it excites quasiparticles with the gap $2|\Delta|$ as well as those with $|\Delta|$. This result might be essential when we consider physics of the Neutron Star.

V. CONCLUSION

In this paper we considered the Andreev reflection of quarks from the QGP/CFL interface. There are two different types of reflection. One is similar to the Andreev reflection from the 2SC color superconductor where one hole (of different color and flavor than incoming particle) is reflected toward the QGP phase. However in the CFL phase there

is also the possibility of the reflection of three holes. This is connected to the fact that in CFL phase there exist two independent Cooper pair structures: one related to octet and one related to singlet representations of $SU(3)$ group. In the basis chosen in our paper the example of such a process is the reflection of holes of u red quark, d green quark and s blue quark from the incoming particle of u red type.

The importance of the Andreev reflection phenomenon for the baryon transport processes is given by the equation (15). From this equation it is seen that the baryonic current is strongly suppressed by the Boltzman factor for energy of incoming quarks which are usually of the order of temperature inside the Nuclear Stars (and $T \ll |\Delta|$). This process is more important for the QGP/CFL interface than for the QGP/2SC because in the case of CFL phase all quarks in all colors are paired. The existence of massless Nambu-Goldstone mode connected to the baryon symmetry breaking, as was shown, does not influence the transport of baryon number through the interface in the leading order.

The interior of the Neutron Star is a complicated and not completely well-known object and in particular can contain many (or single) interfaces. In that situation the dynamics of the baryonic matter propagation would be strongly affected by the process of Andreev reflection. But this subject remains to be done.

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Appendix

Let us defined the bispinors $\varphi_{r,R}(\vec{k})$ and $\varphi_{r,L}(\vec{k})$ through the equations:

$$\begin{aligned} (\alpha \cdot \vec{k} + m\gamma_0 - \mu)\varphi_{r,R}(\vec{k}) &= \epsilon\varphi_{r,R}(\vec{k}) \\ \varphi_{r,L}^\dagger(\vec{k})(\alpha \cdot \vec{k} + m\gamma_0 - \mu) &= \epsilon\varphi_{r,L}^\dagger(\vec{k}) \end{aligned} \quad (16)$$

where r describes the spin and momentum \vec{k} can be in general complex vector. From this reason one has to distinguish between the right- and left-handed eigenvectors, which is denoted by the capital letters L, R . For \vec{k} complex $\varphi_{r,L}^\dagger$ is not hermitian conjugate to $\varphi_{r,R}$. The solution of the equations takes the form:

$$\begin{aligned} \varphi_{r,R}(\vec{k}) &= \begin{pmatrix} \sqrt{m + \epsilon + \mu}\chi_r \\ \frac{\vec{\sigma} \cdot \vec{k}}{\sqrt{m + \epsilon + \mu}}\chi_r \end{pmatrix} \\ \varphi_{r,L}^\dagger(\vec{k}) &= \left(\bar{\chi}_r^\dagger \sqrt{m + \epsilon + \mu}, \bar{\chi}_r^\dagger \frac{\vec{\sigma} \cdot \vec{k}}{\sqrt{m + \epsilon + \mu}} \right) \end{aligned} \quad (17)$$

where $\chi_r, \bar{\chi}_r^\dagger$ are spinors and where $\epsilon = \sqrt{\vec{k}^2 + m^2} - \mu^2$. The spinors can be defined in the helicity basis:

$$\begin{aligned} \vec{\sigma} \cdot \vec{k}\chi_{\uparrow,\downarrow} &= \pm k\chi_{\uparrow,\downarrow} \\ \bar{\chi}_{\uparrow,\downarrow}^\dagger \vec{\sigma} \cdot \vec{k} &= \pm k\bar{\chi}_{\uparrow,\downarrow}^\dagger \end{aligned} \quad (18)$$

where $k = \sqrt{\vec{k}^2}$. Let us also define the additional bispinors:

$$\begin{aligned} h_{r,L}^\dagger(\vec{k})(\alpha \cdot \vec{k} - m\gamma_0 + \mu) &= \bar{\epsilon}h_{r,L}^\dagger(-\vec{k}) \\ (\alpha \cdot \vec{k} - m\gamma_0 + \mu)h_{r,R}(-\vec{k}) &= \bar{\epsilon}h_{r,R}(-\vec{k}) \end{aligned} \quad (19)$$

where $\bar{\epsilon} = -\epsilon$ and bispinors are given by formulae:

$$\begin{aligned} h_{r,R}(-\vec{k}) &= \begin{pmatrix} \sqrt{m - \epsilon + \mu}\chi_r \\ \frac{\vec{\sigma} \cdot \vec{k}}{\sqrt{m - \epsilon + \mu}}\chi_r \end{pmatrix} \\ h_{r,L}^\dagger(-\vec{k}) &= \left(\bar{\chi}_r^\dagger \sqrt{m - \epsilon + \mu}, -\bar{\chi}_r^\dagger \frac{\vec{\sigma} \cdot \vec{k}}{\sqrt{m - \epsilon + \mu}} \right) \end{aligned} \quad (20)$$

²There is another solution with $\epsilon = -\sqrt{\vec{k}^2 + m^2} - \mu$ which is not interesting for our purposes

The above defined bispinors fulfill simple algebraic relations which are useful in the calculations:

$$\begin{aligned}\varphi_{r,L}^\dagger \varphi_{s,R} &= h_{r,L}^\dagger h_{s,R} = 2\sqrt{k^2 + m^2} \delta_{rs} \\ \varphi_{s,L}^\dagger C \gamma_5 h_{r,L}^{\dagger T} h_{s,R} &= \varphi_{s,R}^T C \gamma_5 h_{r,R} = 2\sqrt{k^2 + m^2} \begin{cases} -1 & s=\uparrow \ r=\downarrow \\ 1 & s=\downarrow \ r=\uparrow \end{cases}\end{aligned}\tag{21}$$

The bispinors defined above has simple physical meaning in the QGP phase. The wavefunction:

$$\psi(t, \vec{r}) = \varphi_{\uparrow R}(\vec{k}) \exp(-i\epsilon t + i\vec{k} \cdot \vec{r})\tag{22}$$

describes the particle of spin projection up, velocity $\vec{v} = \frac{\vec{k}}{E}$, where $E = \sqrt{k^2 + m^2}$ and energy ϵ above the Fermi Sea. From the other hand the wavefunction:

$$\psi^\dagger(t, \vec{r}) = h_{\downarrow L}^\dagger(-\vec{k}) \exp(-i\epsilon t + i\vec{k} \cdot \vec{r})\tag{23}$$

describes the hole of spin projection down, velocity $-\vec{v}$, and energy ϵ below the Fermi Sea.

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